Holographic QCD for H-dibaryon (uuddss)

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Abstract. The H-dibaryon (uuddss) is studied in holographic QCD for the first time. In holographic QCD, four-dimensional QCD, i.e., SU(N_c) gauge theory with chiral quarks, can be formulated with S^1-compactified D4/D8/$\overline{\text{D8}}$-brane system. In holographic QCD with large N_c, all the baryons appear as topological chiral solitons of Nambu-Goldstone bosons and (axial) vector mesons, and the H-dibaryon can be described as an SO(3)-type topological soliton with $B = 2$. We derive the low-energy effective theory to describe the H-dibaryon in holographic QCD. The H-dibaryon mass is found to be twice of the $B = 1$ hedgehog-baryon mass, $M_{\rm H} \simeq 2.00 M^{\rm HH}_{B=1}$, and is estimated about 1.7GeV, which is smaller than mass of two nucleons (flavor-octet baryons), in the chiral limit.

1 Introduction

Nowadays, QCD is established as the fundamental theory of the strong interaction, and all the experimentally observable hadrons have been considered as color-singlet composite particles of quarks and gluons. From QCD, as well as ordinary mesons ($\bar{q}q$) and baryons (qqq) in the valence picture, there can exist "exotic hadrons" [1] such as glueballs, multi-quarks [2, 3] and hybrid hadrons, and the exotic-hadron physics has been an interesting field theoretically and experimentally.

The H-dibaryon, $B = 2$ SU(3) flavor-singlet bound state of uuddss, has been one of the oldest multi-quark candidates, first predicted by R. L. Jaffe in 1977 from a group-theoretical argument of the color-magnetic interaction in the MIT bag model [2]. In 1985, the H-dibaryon was also investigated [4, 5] in the Skyrme-Witten model [6–8]. These two model calculations suggested a low-lying H-dibaryon below the ΛΛ threshold, which means the stability of H against the strong decay. In 1991, however, Imai group experimentally excluded the low-lying H-dibaryon [9], and found the first event of the double hyper nuclei, i.e., ${}^{6}_{\Lambda\Lambda}$He, instead. Then, the current interest is mainly possible existence of the H-dibaryon as a resonance state.

Theoretically, it is still interesting to consider the stability of H-dibaryons in the SU(3) flavor-symmetric case of $m_u = m_d = m_s$ [10–12], because the large mass of H may be due to an SU(3) flavor-symmetry breaking by the large s-quark mass, $m_s \gg m_{u,d}$, in the real world. Actually, recent lattice QCD simulations suggest the stable H-dibaryon in an SU(3) flavor-symmetric and large quark-mass region [10, 11].

So, how about the H-dibaryon in the chiral limit of $m_u = m_d = m_s = 0$? Although the lattice QCD calculation is usually a powerful method to evaluate hadron masses, it is fairly difficult to take the chiral limit, because a large-volume lattice is needed for such a calculation to control massless pions.

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In this paper, we study the H-dibaryon and its properties in the chiral limit using holographic QCD [13], which has a direct connection to QCD, unlike most effective models. In particular, we investigate the H-dibaryon mass from the viewpoint of its stability in the chiral limit.

2 Holographic QCD

In this section, we briefly summarize the construction of holographic QCD from a D-brane system [14, 15], and derive the low-energy effective theory of QCD [16] at the leading order of $1/N_c$ and $1/\lambda$ expansions, where the 't Hooft coupling $\lambda \equiv N_c g^2_{\rm YM}$ is given with the gauge coupling $g_{\rm YM}$.

2.1 QCD-equivalent D-brane system

Just after J. M. Maldacena's discovery of the AdS/CFT correspondence in 1997 [17], E. Witten [14] succeeded in 1998 the formulation of non-SUSY four-dimensional pure SU(N_c) gauge theories using an S^1-compactified D4-brane in the superstring theory. In 2005, Sakai and Sugimoto showed a remarkable formulation of four-dimensional QCD, i.e., SU(N_c) gauge theory with chiral quarks, using an S^1-compactified D4/D8/$\overline{\rm D8}$-brane system [15], as shown in Fig. 1. Such a construction of QCD is often called holographic QCD.

This QCD-equivalent D-brane system consists of N_c D4-branes and N_f D8/$\overline{\rm D8}$-branes, which give color and flavor degrees of freedom, respectively. In this system, gluons appear as 4-4 string modes on N_c D4-branes, and the left/right quarks appear as 4-8/4-$\bar{8}$ string modes at the cross point between D4 and D8/$\overline{\rm D8}$ branes, as shown in Fig. 1. This D-brane system possesses the SU(N_c) gauge symmetry and the exact chiral symmetry [15], and gives QCD in the chiral limit.

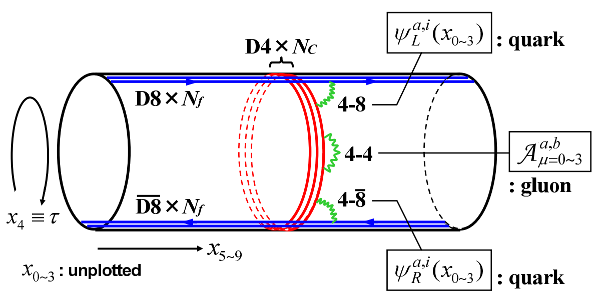

Figure 1. Construction of holographic QCD with an S^1-compactified D4/D8/$\overline{\rm D8}$-brane system, which corresponds to non-SUSY four-dimensional QCD with chiral quarks [15, 16]. This figure is taken from Ref.[16].

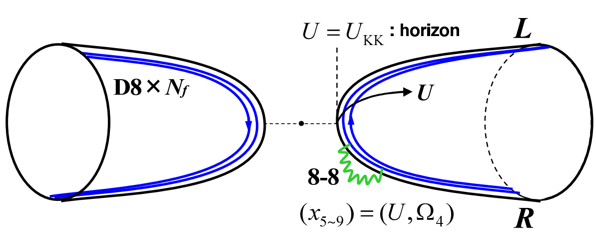

Figure 2. Holographic QCD after the replacement of large-N_c D4 branes by a gravitational background via the gauge/gravity correspondence [14–16]. This figure is taken from Ref.[16].

In holographic QCD, $1/N_c$ and $1/\lambda$ expansions are usually taken. In large N_c, D4-branes are the dominant gravitational source, and can be replaced by their SUGRA solution [15] as shown in Fig. 2, via the gauge/gravity correspondence. In large λ, the strong-coupling gauge theory is converted into a weak-coupling gravitational theory [14]. In this paper, we consider the leading order of $1/N_c$ and $1/\lambda$ expansions.

2.2 Low-energy effective theory

In the presence of the D4-brane gravitational background g_{MN}, the D8/$\overline{\text{D8}}$ brane system can be expressed with the non-Abelian Dirac-Born-Infeld (DBI) action,

$$S_{\mathrm{D8}}^{\mathrm{DBI}} = T_8 \int d^9x\, e^{-\phi} \sqrt{-\det(g_{MN} + 2\pi\alpha' F_{MN})}\,, \tag{1}$$

at the leading order of $1/N_c$ and $1/\lambda$ expansions. Here, $F_{MN} \equiv \partial_M A_N - \partial_N A_M + i[A_M, A_N]$ is the field strength of the U(N_f) gauge field A_M in the flavor space on the D8 brane. The surface tension T_8, the dilaton field ϕ and the Regge slope parameter α' are defined in the framework of the superstring theory, and, for the simple notation, we have taken the $M_{\mathrm{KK}} = 1$ unit, where the Kaluza-Klein mass M_{KK} is the energy scale of this theory [15].

After some calculations, one can derive the meson theory equivalent to infrared QCD at the leading order of $1/N_c$ and $1/\lambda$ [15, 16]. For the construction of the low-energy effective theory, we only consider massless Nambu-Goldstone (NG) bosons and the lightest SU(N_f) vector meson $\rho_\mu(x) \equiv \rho_\mu(x)^a T^a \in \mathrm{su}(N_f)$, which we simply call "$\rho$-meson". We eventually derive the four-dimensional effective action in Euclidean space-time $x^\mu = (t, \mathbf{x})$ [16],

$$\begin{aligned} S_{\mathrm{HQCD}} = \int d^4x \Big\{ & \frac{f_\pi^2}{4}\mathrm{tr}(L_\mu L_\mu) - \frac{1}{32e^2}\mathrm{tr}[L_\mu, L_\nu]^2 + \frac{1}{2}\mathrm{tr}(\partial_\mu\rho_\nu - \partial_\nu\rho_\mu)^2 + m_\rho^2 \mathrm{tr}(\rho_\mu\rho_\mu) \\ & - ig_{3\rho}\mathrm{tr}\{(\partial_\mu\rho_\nu - \partial_\nu\rho_\mu)[\rho_\mu, \rho_\nu]\} - \frac{1}{2}g_{4\rho}\mathrm{tr}[\rho_\mu, \rho_\nu]^2 + ig_1\mathrm{tr}\{[\alpha_\mu, \alpha_\nu](\partial_\mu\rho_\nu - \partial_\nu\rho_\mu)\} \\ & + g_2\mathrm{tr}\{[\alpha_\mu, \alpha_\nu][\rho_\mu, \rho_\nu]\} + g_3\mathrm{tr}\{[\alpha_\mu, \alpha_\nu]([\beta_\mu, \rho_\nu] + [\rho_\mu, \beta_\nu])\} \\ & - ig_4\mathrm{tr}\{(\partial_\mu\rho_\nu - \partial_\nu\rho_\mu)([\beta_\mu, \rho_\nu] + [\rho_\mu, \beta_\nu])\} - g_5\mathrm{tr}\{[\rho_\mu, \rho_\nu]([\beta_\mu, \rho_\nu] + [\rho_\mu, \beta_\nu])\} \\ & - \frac{1}{2}g_6\mathrm{tr}([\alpha_\mu, \rho_\nu] + [\rho_\mu, \alpha_\nu])^2 - \frac{1}{2}g_7\mathrm{tr}([\beta_\mu, \rho_\nu] + [\rho_\mu, \beta_\nu])^2 \Big\}, \end{aligned} \tag{2}$$

where L_μ is defined with the chiral field $U(x)$ or the NG boson field $\pi(x) \equiv \pi^a(x)T^a \in \mathrm{su}(N_f)$ as

$$L_\mu \equiv \frac{1}{i}U^\dagger \partial_\mu U \in \mathrm{su}(N_f), \quad U(x) \equiv e^{i2\pi(x)/f_\pi} \in \mathrm{SU}(N_f). \tag{3}$$

The axial vector current α_μ and the vector current β_μ are defined as

$$\alpha_\mu \equiv l_\mu - r_\mu \in \mathrm{su}(N_f)_A, \quad \beta_\mu \equiv \frac{1}{2}(l_\mu + r_\mu) \in \mathrm{su}(N_f)_V, \tag{4}$$

with the left and the right currents,

$$l_\mu \equiv \frac{1}{i}\xi^\dagger \partial_\mu \xi, \quad r_\mu \equiv \frac{1}{i}\xi\partial_\mu\xi^\dagger, \quad \xi(x) \equiv e^{i\pi(x)/f_\pi} \in \mathrm{SU}(N_f). \tag{5}$$

Thus, we obtain the effective meson theory derived from QCD in the chiral limit at the leading order of $1/N_c$ and $1/\lambda$ expansions. Note that this theory has just two independent parameters, e.g., the Kaluza-Klein mass $M_{\mathrm{KK}} \sim 1$GeV and $\kappa \equiv \lambda N_c/216\pi^3$ [15, 18], and all the coupling constants and masses in the effective action (2) are expressed with them [16]. As a remarkable fact, in the absence of the ρ-meson, this effective theory reduces to the Skyrme-Witten model [6] in Euclidean space-time,

$$\mathcal{L}_{\mathrm{Skyrme}} = \frac{f_\pi^2}{4}\mathrm{tr}(L_\mu L_\mu) - \frac{1}{32e^2}\mathrm{tr}[L_\mu, L_\nu]^2. \tag{6}$$

3 H-dibaryon as a B=2 Topological Chiral Soliton in Holographic QCD

As a general argument, large-N_c, QCD becomes a weakly interacting meson theory, and baryons are described as topological chiral solitons of mesons [7]. In holographic QCD with large N_c, the H-dibaryon is also described as a $B = 2$ chiral soliton, and its static profile is expressed with the "SO(3)-type hedgehog Ansatz", similarly in the Skyrme-Witten model [4, 5]. Here, the SO(3) is the flavor-symmetric subalgebra of $\mathrm{SU}(3)_f$, and its generators $\Lambda_{i=1,2,3}$ are

$$\Lambda_1 = \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \Lambda_2 = -\lambda_5 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \quad \Lambda_3 = \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{7}$$

which satisfy the SO(3) algebra and the following relations,

$$[\Lambda_i, \Lambda_j] = i\epsilon_{ijk}\Lambda_k, \quad (\mathbf{\Lambda}\cdot\hat{\mathbf{x}})^3 = \mathbf{\Lambda}\cdot\hat{\mathbf{x}}, \quad \mathrm{Tr}[(\mathbf{\Lambda}\cdot\hat{\mathbf{x}})^2 - 2/3] = 0, \tag{8}$$

with $\hat{\mathbf{x}} \equiv \mathbf{x}/r$ and $r \equiv |\mathbf{x}|$. The SO(3)-type hedgehog Ansatz [4, 5, 13] is generally expressed as

$$U(\mathbf{x}) = e^{i\{(\mathbf{\Lambda}\cdot\hat{\mathbf{x}})F(r)+[(\mathbf{\Lambda}\cdot\hat{\mathbf{x}})^2-2/3]\varphi(r)\}} \in \mathrm{SU}(3)_f, \quad F(r) \in \mathbf{R}, \quad \varphi(r) \in \mathbf{R}, \tag{9}$$

where $F(r)$ and $\varphi(r)$ are the chiral profile functions characterizing the NG boson field. Note that $U(\mathbf{x})$ in Eq.(9) is the general form of the special unitary matrix which consists of $\mathbf{\Lambda}\cdot\hat{\mathbf{x}}$, because of Eq.(8). For the topological soliton, the $B = 2$ boundary condition [4, 5] is given as

$$F(\infty) = \varphi(\infty) = 0, \quad F(0) = \varphi(0) = \pi. \tag{10}$$

On the $\mathrm{SU}(3)_f$ ρ-meson field, we use the SO(3) Wu-Yang-'t Hooft-Polyakov Ansatz,

$$\rho_0(\mathbf{x}) = 0, \quad \rho_i(\mathbf{x}) = \epsilon_{ijk}\hat{x}_j G(r)\Lambda_k \in \mathrm{so}(3) \subset \mathrm{su}(3), \quad G(r) \in \mathbf{R}, \tag{11}$$

similarly in the $B = 1$ case in holographic QCD [16]. (This $G(r)$ corresponds to $-\tilde{G}(r)$ in Ref.[16].) Thus, all the above treatments are symmetric in the (u, d, s) flavor space.

Substituting Ansätze (9) and (11) in Eq.(2), we derive the effective action to describe the static H-dibaryon in terms of the profile functions $F(r)$, $\varphi(r)$ and $G(r)$ [13]:

$$\begin{aligned}
S_{\mathrm{HQCD}} = \int d^4x \Big\{ & \frac{f_\pi^2}{4}\Big[\frac{2}{3}\varphi'^2 + 2F'^2 + \frac{8}{r^2}(1-\cos F\cos\varphi)\Big] + \frac{1}{32e^2}\frac{16}{r^2}\Big[(\varphi'^2 + F'^2)(1-\cos F\cos\varphi) \\
& + 2\varphi' F'\sin F\sin\varphi + \frac{1}{r^2}\{(1-\cos F\cos\varphi)^2 + 3\sin^2 F\sin^2\varphi\}\Big] \\
& + \frac{1}{2}\Big[8\Big(\frac{3}{r^2}G^2 + \frac{2}{r}GG' + G'^2\Big)\Big] + m_\rho^2[4G^2] + g_{3\rho}\Big[8\frac{G^3}{r}\Big] + \frac{1}{2}g_{4\rho}[4G^4] \\
& - g_1\Big[\frac{16}{r}\Big\{\Big(\frac{1}{r}G + G'\Big)\Big(F'\sin\frac{F}{2}\cos\frac{\varphi}{2} + \varphi'\cos\frac{F}{2}\sin\frac{\varphi}{2}\Big) + \frac{1}{r^2}G(1-\cos F\cos\varphi)\Big\}\Big] \\
& - g_2\Big[\frac{8}{r^2}G^2(1-\cos F\cos\varphi)\Big] \\
& + g_3\Big[\frac{16}{r^3}G\Big\{3\sin F\sin\frac{F}{2}\sin\varphi\sin\frac{\varphi}{2} + \Big(1-\cos\frac{F}{2}\cos\frac{\varphi}{2}\Big)(1-\cos F\cos\varphi)\Big\}\Big] \\
& - g_4\Big[\frac{16}{r^2}G^2\Big(1-\cos\frac{F}{2}\cos\frac{\varphi}{2}\Big)\Big] - g_5\Big[\frac{8}{r}G^3\Big(1-\cos\frac{F}{2}\cos\frac{\varphi}{2}\Big)\Big] \\
& + g_6[4G^2(F'^2 + \varphi'^2)] + g_7\Big[\frac{8}{r^2}G^2\Big\{3\sin^2\frac{F}{2}\sin^2\frac{\varphi}{2} + \Big(1-\cos\frac{F}{2}\cos\frac{\varphi}{2}\Big)^2\Big\}\Big]\Big\} \\
= \int dt \int_0^\infty & dr\, 4\pi r^2 \varepsilon[F(r), \varphi(r), G(r)].
\end{aligned} \tag{12}$$

4 H-dibaryon Solution in Holographic QCD

To obtain the topological soliton solution of the H-dibaryon in holographic QCD, we numerically calculate the profiles $F(r)$, $\varphi(r)$ and $G(r)$ [13] by minimizing the Euclidean effective action (12) under the boundary condition (10) [19]. The two independent parameters, e.g., $M_{\rm KK}$ and $\kappa \equiv \lambda N_c/216\pi^3$, are set to reproduce the pion decay constant f_π=92.4MeV and the ρ-meson mass m_ρ=776MeV [15, 16].

For the H-dibaryon solution in holographic QCD, we obtain the chiral profiles, $F(r)$ and $\varphi(r)$, and the scaled ρ-meson profile $G(r)/\kappa^{1/2}$ as shown in Fig. 3, and estimate the H-dibaryon mass of $M_{\rm H} \simeq 1673$MeV in the chiral limit. Figure 4 shows the energy density $4\pi r^2\varepsilon(r)$ in the H-dibaryon. The root mean square radius of the H-dibaryon is estimated as $\sqrt{\langle r^2\rangle_{\rm H}} \simeq 0.413$fm in terms of the energy density. For comparison, we calculate the $B = 1$ hedgehog (HH) baryon in holographic QCD with the same numerical condition, and estimate $M_{B=1}^{\rm HH} \simeq 836.7$MeV and $\sqrt{\langle r^2\rangle_{B=1}^{\rm HH}} \simeq 0.362$fm. Thus, the H-dibaryon mass is twice of the $B = 1$ hedgehog-baryon mass, $M_{\rm H} \simeq 2.00 M_{B=1}^{\rm HH}$.

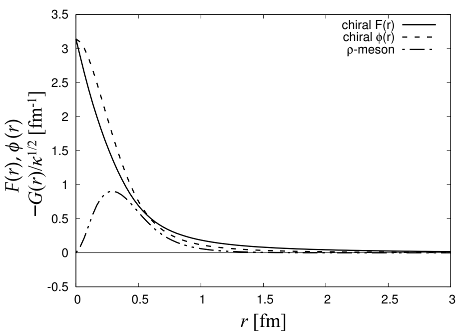

Figure 3. The chiral profiles, $F(r)$ and $\varphi(r)$, and the scaled ρ-meson profile $G(r)/\kappa^{1/2}$ in the H-dibaryon as the SO(3)-type hedgehog soliton solution in holographic QCD. Here, the topological boundary condition of $B = 2$ is $F(0) = \varphi(0) = \pi$ and $F(\infty) = \varphi(\infty) = 0$.

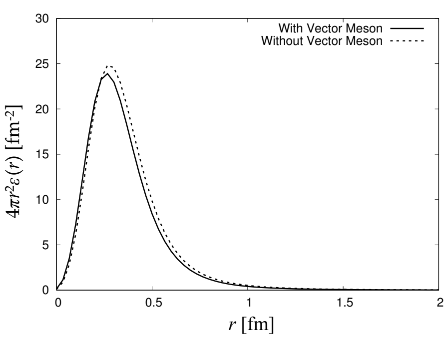

Figure 4. The energy density distribution $4\pi r^2\varepsilon(r)$ in the H-dibaryon (solid curve), and that without vector mesons (dashed curve) for comparison.

We summarize in Table 1 the mass and the radius of the H-dibaryon and the $B = 1$ hedgehog baryon in holographic QCD. Since the nucleon mass $M_{\rm N}$ is larger than the $B = 1$ hedgehog mass $M_{B=1}^{\rm HH}$ by the rotational energy [6, 8], the H-dibaryon mass is smaller than mass of two nucleons (flavor-octet baryons) , $M_{\rm H} < 2M_{\rm N}$, in the chiral limit.

Finally, we examine the vector-meson effect for the H-dibaryon by comparing with the $\rho(x) = 0$ case. As the result, we find that the chiral profiles $F(r)$ and $\varphi(r)$ are almost unchanged and slightly shrink by the vector-meson effect, and the energy density also shrinks slightly, as shown in Fig. 4.

Table 1. The mass $M_{\rm H}$ and the radius $\sqrt{\langle r^2\rangle_{\rm H}}$ of the H-dibaryon in the chiral limit in holographic QCD, together with those of the $B=1$ hedgehog (HH) baryon.

$M_{\rm H}$	$\sqrt{\langle r^2\rangle_{\rm H}}$	$M_{B=1}^{\rm HH}$	$\sqrt{\langle r^2\rangle_{B=1}^{\rm HH}}$
1673 MeV	0.413 fm	836.7 MeV	0.362 fm

As a significant vector-meson effect, we find that about 100MeV mass reduction is caused by the interaction between NG bosons and vector mesons in the interior region of the H-dibaryon.

5 Summary and Concluding Remarks

We have studied the H-dibaryon (uuddss) as the $B=2$ SO(3)-type topological chiral soliton solution in holographic QCD for the first time. The H-dibaryon mass is twice of the $B=1$ hedgehog-baryon mass, $M_{\rm H} \simeq 2.00 M_{B=1}^{\rm HH}$, and is estimated about 1.7GeV, which is smaller than mass of two nucleons (flavor-octet baryons), in the chiral limit. In holographic QCD, we have found that the vector-meson effect gives a slight shrinkage of the chiral profiles and the energy density, and also gives about 100MeV mass reduction of the H-dibaryon.

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